Environmental tipping points significantly affect the cost–benefit assessment of climate policies

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Most current cost–benefit analyses of climate change policies suggest an optimal global climate policy that is significantly less stringent than the level required to meet the internationally agreed 2 °C target. This is partly because the sum of estimated economic damage of climate change across various sectors, such as energy use and changes in agricultural production, results in only a small economic loss or even a small economic gain in the gross world product under predicted levels of climate change. However, those cost–benefit analyses rarely take account of environmental tipping points leading to abrupt and irreversible impacts on market and nonmarket goods and services, including those provided by the climate and by ecosystems. Here we show that including environmental tipping point impacts in a stochastic dynamic integrated assessment model profoundly alters cost–benefit assessment of global climate policy. The risk of a tipping point, even if it only has nonmarket impacts, could substantially increase the present optimal carbon tax. For example, a risk of only 5% loss in nonmarket goods that occurs with a 5% annual probability at 4 °C increase of the global surface temperature causes an immediate two-thirds increase in optimal carbon tax. If the tipping point also has a 5% impact on market goods, the optimal carbon tax increases by more than a factor of 3. Hence existing cost–benefit assessments of global climate policy may be significantly underestimating the needs for controlling climate change.

climate change | tipping point | ecosystem | carbon tax |
relative price effect

Tipping points in the climate system (1) and in ecosystems (2, 3) could be crossed in a changing climate. The resulting impacts are expected to reduce the environmental goods and services provided to humanity by the climate and by ecosystems (4). Some of those impacts will be on goods that have direct market value, such as the food produced from agricultural ecosystems. Other impacts will be on services that do not involve any production processes of market goods but can still directly affect human well-being through, e.g., health effects, changes in physical comfort, sensory satisfaction, or spiritual fulfillment—making them nonmarket impacts (5).

Environmental tipping points can occur at a range of spatial scales (6), from global-scale tipping points in the climate system, such as a reorganization of the Atlantic Meridional Overturning Circulation (1, 7), to ecosystem-scale tipping points, such as sudden lake eutrophication (2). Here we consider the idealized case of an instantaneous tipping point that occurs on a sufficient scale to impact the global economy. Such a tipping point could come from a physical tipping element in the climate system, such as the West African or Indian monsoons (1), which in turn impacts humans and ecosystems, or it could come from a more biological tipping element such as a major biome (1). For example, widespread dieback of forests has been observed in Canada (8, 9), both boreal and tropical biomes are thought to exhibit multiple stable states (10–12), and abrupt forest dieback has been forecast in both the Amazon and boreal regions in future (1, 7). There is even speculation that an abrupt and irreversible shift of ecosystems could occur on a planetary scale (3, 4). Whether they themselves tip or they are impacted by tipping in a more physical part of the climate system, ecosystems and the goods and services they provide carry significant market and nonmarket values (5) (as presumably do the goods and services provided by more physical parts of the climate system).

Predicting when tipping points will occur is inherently uncertain (1, 2), because they occur in imperfectly understood complex systems, which are subject to stochastic environmental variability (as well as deterministic forcing), meaning that their time of tipping can never be forecast precisely (13). The representation of such risk and uncertainty is recognized as an unresolved issue for the estimation of the social cost of carbon (14–16).

Here we explore how the risk of stochastically uncertain environmental tipping points that have nonmarket, or both market and nonmarket, impacts affects the cost–benefit assessment of climate change policies. The majority of attempts to assess the economic implications of the impacts of climate change concentrate on market impacts, whose estimation can draw information from market statistics (approaches and limitations of which are discussed in, e.g., refs. 15 and 17). Most integrated assessment models (IAMs) also include nonmarket impacts, but they tend to discount these future impacts without accounting for increases in their relative price as environmental goods and services become scarcer (18, 19). The damages in IAMs are also often smooth functions of temperature that do not account for abrupt and irreversible impacts from tipping points. Finally, many IAMs are deterministic, failing to consider uncertainty surrounding the impacts of climate change.

Each of these three weaknesses has been addressed individually in existing studies. The limited substitutability of ecosystem services has been shown to increase the welfare impact of these nonmarket losses, as discussed by Hoel and Sterner (18) and Sterner and Persson (19). The prospect of irreversible,

Significance

Most current cost–benefit analyses of climate change suggest global climate policy should be relatively weak. However, relatively few studies account for the market or nonmarket impacts of passing environmental tipping points that cause abrupt and irreversible damages. We use a stochastic dynamic model of the climate and economy to quantify the effect of tipping points on climate change policy. We show that environmental tipping points can profoundly alter cost–benefit analysis, justifying a much more stringent climate policy, which takes the form of a higher immediate price on carbon.

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environmental tipping points has been shown to produce a precautionary optimal management response in many cases (20–22). Stochastic uncertainty surrounding climate change damages has been shown to generally increase the optimal level of mitigation (23, 24). Furthermore, the combination of stochastic uncertainty and abrupt, irreversible patterns of climate change has been shown to increase optimal levels of mitigation (25, 26). Here, we address the three issues simultaneously, analyzing how the stochastic component of climate change risk interacts with the limited substitutability of environmental goods and services, under irreversible tipping.

Approach

For this study, we use a version of the Dynamic Stochastic Integration of Climate and Economy (DSICE) (27), a stochastic IAM of climate and the economy that draws on the Dynamic Integrated Model of Climate and Economy (DICE) 2007 model (28). The DICE model has been used for a variety of questions regarding climate change and policy and has been scrutinized from a number of perspectives. Our results can therefore be readily compared with previous findings. The distinctive feature of DSICE is that it can compute optimal forward-looking decisions by a risk-averse economic agent (social planner) about controlling climate change in the face of uncertainty. Such computations require optimization of decisions at every time point for all ways of resolving uncertainty in the future. Relatively few IAMs have conducted optimization for such an entire sequence of decisions, e.g., refs. 24–26 and 29–33. The model analysis that this study conducts is only possible with such a complete set of optimization.

Here, we only provide the relevant modifications to DSICE (see *Methods* and *SI Text* for a full model description). The original DICE 2007 model (28) takes into account nonmarket impacts, which are an implicit component of the aggregate climate change damage specified by its damage function. However, these are only deterministic impacts and also do not consider limited substitutability between nonmarket and market goods. Here, the benefits of consuming market and nonmarket goods are represented by the following global welfare function, based on refs. 18 and 19:

$$u(C_t, S_t) = \left[(1-\gamma)\left(\frac{C_t}{L_t}\right)^{1-1/\sigma} + \gamma\left(\frac{S_t}{L_t}\right)^{1-1/\sigma}\right]^{(1-\alpha)\sigma/(\sigma-1)} \frac{L_t}{1-\alpha} \qquad \textbf{[1]}$$

where C_t is the global consumption of market goods and S_t are the global nonmarket benefits at time t, L_t is the exogenous population given in the DICE model, σ represents substitutability (the elasticity of substitution) between the market and nonmarket goods, α is the risk aversion parameter (set to be 2, as in the DICE model), and γ is the share of nonmarket benefits in the welfare. With $\sigma \to \infty$, the relationship between C and S would be approximated to be additive (substitutable with each other), and with $\sigma \to 0$, the two goods would lose substitutability and become complementary. The standard DICE model corresponds to the case of $\sigma = 1$ as its damage of climate change is represented with a multiplicative term to consumption. A weak substitutability between C and S is a realistic case because some environmental services are not easily substituted with human-made products. For example, purification of drinking water would become extremely costly if natural processes of forests were fully replaced with artificial processes (34). Just as in many other IAMs, C grows as investment increases capital and output, and is reduced as climate change negatively affects the production processes.

We examine cases in which just S, or both S and C, is subject to loss that increases with the degree of climate change, $T_t^{\mathrm{AT}} - T_0^{\mathrm{AT}}$, where T_t^{AT} is the increase of the global average surface temperature from 1900 and T_0^{AT} denotes the degree of global warming in the first decision period of the model (which, in line with the DICE model, is the year 2005). There is little empirical information about the elasticity of substitution σ (35), and thus we set the benchmark value at $\sigma = 0.5$ in line with an earlier study by Sterner and Persson (19) that introduced the formulation. We also set γ at a modest level of 0.02.

The tipping point risk is represented as an abrupt and permanent loss of welfare, based on a probability distribution that is dependent on the contemporary atmospheric temperature. For the case of a tipping point risk to nonmarket benefits, we assume

$$S_t = \frac{S_0\left(1 - I_t J_S^*\right)}{1 + \lambda\left(T_t^{\mathrm{AT}} - T_0^{\mathrm{AT}}\right)^2} \qquad \textbf{[2]}$$

where $I_t J_S^*$ represents the rate of loss of the tipping event ($I_t = 0$ before the tipping event occurs, and $I_t = 1$ after it), λ is a constant ($\lambda = 0.0034$), and $S_0 = 4.2$ is the level of nonmarket services in the initial model period. [It is calibrated to be only about 10% of the level of consumption of market goods in the initial period, which is on the low end of the levels suggested by previous studies (18, 19, 36).] The level of λ is set by following the logic used by Sterner and Persson (19). For the benchmark case, we set the tipping point hazard rate to be zero when $T_t^{\mathrm{AT}} \le 1$ °C and then linearly increase with T_t^{AT} to become 5% annually at $T_t^{\mathrm{AT}} = 4$ °C (we denote this probability of tipping by P^*, i.e., $P^* = 5\%$). After a tipping point is crossed, S is reduced by 5% (we denote this rate of loss by J_S^*, i.e., $J_S^* = 5\%$). For the case of a tipping point risk to market and nonmarket benefits, we add a tipping point risk on the market goods, using an equivalent formulation for C_t to Eq. **2** with the rate of loss denoted by J_Y^* and the probabilistic loss occurring simultaneously with that on S.

Our simulation results should be taken only as a numerical illustration rather than as a solid prediction grounded on ample scientific evidence. First, we consider the idealized case of a tipping point that only has nonmarket impacts, i.e., setting $J_Y^* = 0$. This allows us to isolate the interaction between stochastic tipping and imperfect substitutability of environmental goods and services. Then we consider the (arguably more realistic) case of

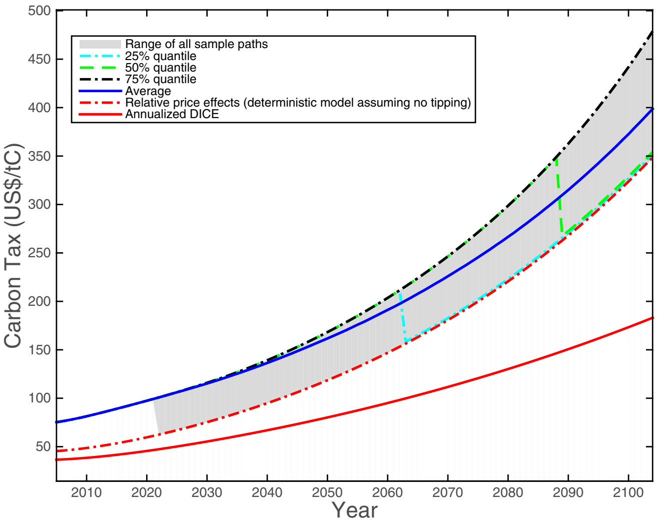

Fig. 1. The optimal carbon taxes (in US dollars per metric ton of carbon) for the case in which the tipping point only impacts nonmarket goods: $\sigma = 0.5$, $\gamma = 0.02$, $P^* = 5\%$, $J_S^* = 5\%$, and $J_Y^* = 0\%$ (see *Approach* for parameter definitions). The state of the environmental good crosses a tipping point probabilistically, and so the paths of 25%, 50%, and 75% quantiles and of the average as well as the range of all sample paths and two reference paths (the path of the annualized DICE model and the path with relative price effects from the deterministic model assuming no tipping).

Table 1. Initial year carbon tax (US dollars per metric ton of carbon) for various cases in which the tipping event causes damages on the nonmarket environmental services

σ	γ	Carbon tax, relative price effects (assuming no tipping)	P^*, percent	J_S^*, percent	Carbon tax, relative price effects with tipping
0.4	0.01	50	5	5	133
	0.02	62	3	5	222
	0.02	62	5	5	342
0.5	0.01	41	5	5	55
	0.02	**45**	3	5	69
	0.02	**45**	3	20	223
	0.02	**45**	5	5	**75**
	0.02	**45**	5	10	112
	0.02	**45**	5	20	378
	0.02	**45**	10	5	83
	0.03	50	3	5	89
	0.03	50	5	5	98
0.75	0.02	38	5	5	42
	0.02	38	5	20	53
	0.03	39	5	5	46
1	0.02	37	5	5	38
	0.02	37	5	20	42
	0.03	38	5	5	39

The initial year carbon tax is US$37/tC for the deterministic DICE model with annual time steps (corresponding to the solid red path on Fig. 1). Bold numbers are the values for the case with $\sigma = 0.5$, $\gamma = 0.02$, $P^* = 5\%$, $J_S^* = 5\%$, and $J_Y^* = 0$.

a tipping point risk that additionally impacts market goods, with $J_Y^* = 5\%$. Because market goods contribute the dominant share $(1 - \gamma)$ of global welfare, this has a much larger direct effect, but it is substitutable.

Although quantitative information on environmental tipping points is still scarce, these parameter levels are likely to be conservative. For example, the latest Intergovernmental Panel on Climate Change Assessment Report (ref. 37, figure 1 in box SPM.1) suggests that climate change poses very high additional risk to unique and threatened ecosystems with a warming of 2 °C and above. A 5% loss in nonmarket or market benefits also seems conservative if we consider impacts on ecosystem services. Already, 3.5–16% of the global value of ecosystem services is estimated to have been lost over 1997–2011, just due to land use change (5). Roughly 20% of the global value of ecosystem services is estimated to be in tidal marsh and mangrove wetlands (5), which may be abruptly lost if rates of sea-level rise exceed rates of sediment surface elevation (38). Coral reefs hold roughly 8% of global ecosystem value, which may already have abruptly declined (5); they are known to have exhibited abrupt shifts linked to ocean warming (2) and are also vulnerable to ocean acidification in the future (39). Temperate and boreal forests hold roughly 7.5% of global ecosystem value (5) and are vulnerable to climate-driven abrupt dieback (8–10). Tropical forests hold roughly 5% of global ecosystem value (5), and large areas could be vulnerable to abrupt dieback (11, 12).

Results

Nonmarket Impacts Only. In Fig. 1, we plot two reference time paths, one with original DICE settings and annual time steps (annualized DICE) and the other with the relative price effect of S but without tipping point risk [relative price effects (deterministic model assuming no tipping)] (19), which increases the initial carbon tax from US$37/tC (per metric ton of carbon) to US$45/tC. The difference between the two paths underscores the relevance of weakly substitutable environmental services on the optimal climate policy, as indicated by ref. 19. Additionally including the risk of abrupt ecosystem change substantially raises the carbon tax in the initial year from US$45/tC to US$75/tC (Fig. 1; see also Table 1 for the values for the initial year). The effects of stochastic tipping point risk on the carbon tax are greatest in the early years (Fig. 1), which is pertinent given that policy debates are naturally focused on the present and what level the present social cost of carbon should be.

Over the course of the time horizon, the economy crosses a tipping point with a probability that increases with the degree of climate change (the cumulative probability at 2100 is 60.1% for the temperature path with no tipping event before 2100, 63.4% for the deterministic "Relative prices effect" temperature path, and 66.0% for the "Annualized DICE" temperature path). After a tipping point is crossed, the level of the optimal carbon tax is reduced because the reduction of S after having crossed a tipping point cannot be reversed by any level of emission control. Because crossing a tipping point is a probabilistic event, the drop of the tax level also occurs probabilistically. Without the tipping point risk, the time trend of the optimal carbon tax replicates a general pattern found in the standard DICE model, that is, the optimal carbon tax gradually rises. The tipping point risk generally increases the tax level before a tipping point. Fig. 2 shows the increases of the global surface temperature from the preindustrial time corresponding to the paths shown in Fig. 1. Overall, the tipping point risk leads to slightly less temperature increase as a result of greater emission control.

Table 1 shows sensitivity runs for the initial year optimal carbon tax rates with different levels of key parameters. The impacts of tipping point risk are qualitatively similar across most of the cases with different levels of parameters, namely, the share of the environmental good in the utility function (γ), the tipping point loss rate (J_S^*), and the probability of tipping at 4 °C temperature increase (P^*). We see that lower σ, higher γ, higher J_S^*, or higher P^* asks for additional carbon tax in the initial year to delay the tipping event. The effect of the tipping point risk becomes very acute (i.e., an initial carbon tax of over $300/tC) either with a lower level of the elasticity parameter ($\sigma = 0.4$) or

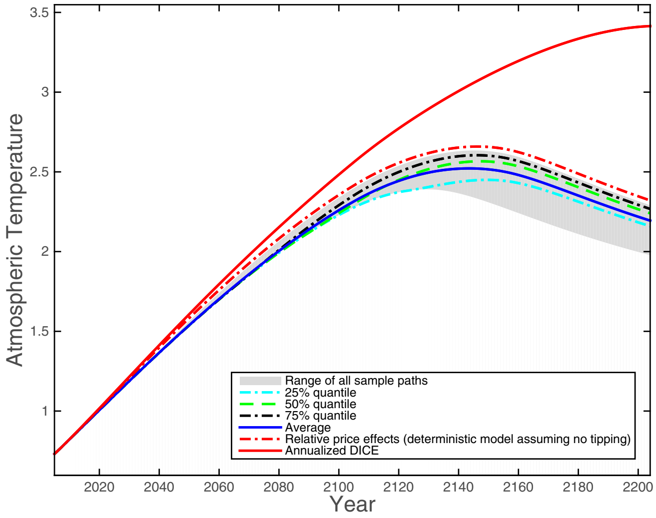

Fig. 2. The increases of the global surface temperature from the preindustrial time for the case in which the tipping point only impacts nonmarket goods. The paths of quantiles and the average are shown as well as the range of all sample paths and two reference paths (the path of the annualized DICE model and the path with relative price effects from the deterministic model assuming no tipping).

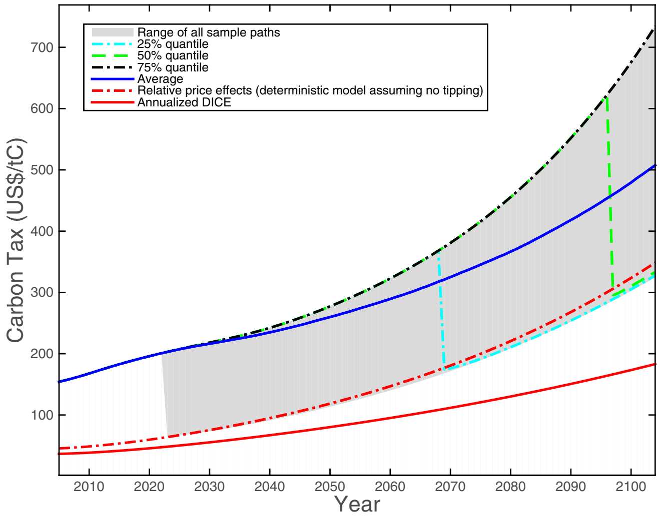

Fig. 3. The optimal carbon taxes (in US dollars per metric ton of carbon) for the case in which the tipping point impacts market and nonmarket goods: $\sigma = 0.5$, $\gamma = 0.02$, $P^* = 5\%$, $J_S^* = 5\%$, and $J_Y^* = 5\%$ (see *Approach* for parameter definitions). The state of the environmental good crosses a tipping point probabilistically, and so the paths of 25%, 50%, and 75% quantiles and of the average are shown as well as the range of all sample paths and two reference paths (the path of the annualized DICE model and the path with relative price effects from the deterministic model assuming no tipping).

with a higher level of the tipping point loss rate ($J_S^* = 20\%$), while the rest of the parameters are kept at the benchmark levels.

However, the effect of tipping point risk almost vanishes with a unitary elasticity of substitution ($\sigma = 1$) when γ, J_S^*, and P^* are at the benchmark levels (i.e., $\gamma = 0.02$, $P^* = 5\%$, and $J_S^* = 5\%$), only raising the initial carbon tax from US$37/tC to US$38/tC (Table 1). This reflects the small share of the environmental good in the utility function, where, even if tipping occurs, it only has a 5% impact on 2% of total welfare—i.e., a 0.1% impact on total welfare. Viewed this way, an increase in carbon price from US$38/tC to US$75/tC can be attributed to introducing weak substitutability of environmental services ($\sigma = 0.5$) (Table 1). Significant effects due to the tipping point risk start to reappear either with an increased fractional impact of a tipping event ($J_S^* = 20\%$ at $\sigma = 1$) or with a slightly reduced level of the elasticity parameter ($\sigma = 0.75$ at $J_S^* = 5\%$). The results with varied levels of σ are consistent with an earlier deterministic study on limited between-good substitutability (40), in that the welfare becomes more strongly limited by the environmental service stream with a lower elasticity of substitution.

Market and Nonmarket Impacts. Figs. 3 and 4 present the results for the case of market and nonmarket impacts from crossing the tipping point ($J_Y^* = J_S^* = 5\%$). Here, introducing the possibility of a future stochastic tipping point increases the initial carbon tax by more than a factor of 3 from US$45/tC (in the deterministic case with $\sigma = 0.5$) to US$154/tC (Table 2). The carbon tax remains persistently circa US$100 above the case with imperfectly substitutable nonmarket goods but no tipping point. The effect of tipping point risk on the carbon tax, compared with that of the relative price effect, is now large not only in the early years but also later in the century. The resulting extra mitigation due to the tipping point risk markedly reduces the average optimal temperature profile (Fig. 4), compared with the temperature path with the relative price effect only.

Table 2 presents the corresponding sensitivity runs, including an environmental tipping risk on market benefits (i.e., $J_Y^* > 0$) as well as nonmarket benefits ($J_S^* > 0$). The levels of the initial year carbon tax in Table 2 are higher than those of the corresponding cases in Table 1, because of the additional damages. Even with a unitary elasticity of substitution and the benchmark levels of the other parameters, there is now a large effect from the tipping point risk, raising the initial carbon tax from US$37/tC to US$111/tC (because the tipping point risk now threatens *ca.* 5% of total welfare). Still, contrasting Tables 1 and 2 reveals that even if the share of the nonmarket goods is small, the relative effects of the nonmarket tipping risk are significant with imperfect substitutability. For example, with $\sigma = 0.5$, $\gamma = 0.02$, and $P^* = 5\%$, the tipping point risk of a 20% loss on the nonmarket goods only ($J_S^* = 20\%$) and that of a 10% loss on both the market and nonmarket goods ($J_S^* = 10\%$ and $J_Y^* = 10\%$) give nearly equal levels of the initial year carbon tax (US$378/tC and US$365/tC, respectively).

Discussion

Two factors are at play for such strong effects of environmental tipping point risk on the optimal climate policy. One is the weak substitutability between goods. The basic idea is described elsewhere (18, 19, 36, 41, 42) and summarized as follows: Even a modest decline of environmental services by climate change can have significant impact on the utility if they are not easily substitutable with others as input for the utility. This is exacerbated by the so-called Baumol's disease, which is when the amount of environmental goods does not grow as much as the economy grows. The other factor is the significance of risk aversion effects for a tipping point risk. Humans are typically risk averse. For a risk-averse individual, it can be rational to pay a significant premium to avoid even a small risk—an example, besides climate change, is fire insurance. In the context of emission control decisions, this means that the economy should significantly increase emission reduction at present to avoid environmental tipping point risk in the future.

The significance of risk aversion for the dynamics is consistent with previous modeling studies of tipping point risks with perfect substitutability (25, 26, 42). General patterns of the time profiles of our results are also consistent with those of previous studies based on simpler settings of renewable resource modeling (ref. 20

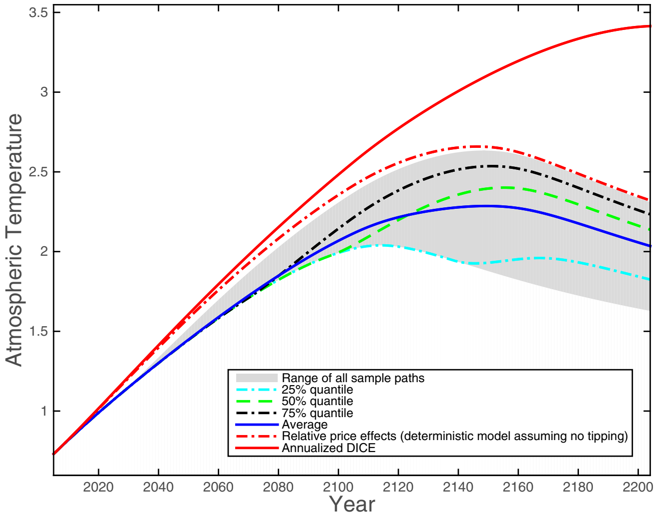

Fig. 4. The increases of the global surface temperature from the preindustrial time for the case in which the tipping point impacts market and nonmarket goods. The paths of quantiles and the average are shown as well as the range of all sample paths and two reference paths (the path of the annualized DICE model and the path with relative price effects from the deterministic model assuming no tipping).

Table 2. Initial year carbon tax (US dollars per metric ton of carbon) for various cases in which the tipping event causes damages on the nonmarket environmental services and the market goods

σ	γ	Carbon tax, relative price effects (assuming no tipping)	P^*, percent	J_S^*, percent	J_Y^*, percent	Carbon tax, relative price effects with tipping
0.5	0.02	**45**	3	5	5	124
	0.02	**45**	3	5	10	191
	0.02	**45**	5	5	5	**154**
	0.02	**45**	5	5	10	267
	0.02	**45**	5	10	5	215
	0.02	**45**	5	10	10	365
	0.03	50	5	5	5	185
	0.03	50	5	5	10	318
1	0.02	37	5	5	5	111
	0.02	37	5	5	10	201

The initial year carbon tax is US$37/tC for the deterministic DICE model with annual time steps (corresponding to the solid red path on Fig. 3). Bold numbers are the values for the case with $\sigma = 0.5$, $\gamma = 0.02$, $P^* = 5\%$, $J_S^* = 5\%$, and $J_Y^* = 5\%$.

and references therein), which show that precautionary actions enhance welfare if a regime shift is determined with an endogenous hazard rate and does not involve a total collapse of the environmental asset (as in our case). What is interesting about our results is that the combination of stochastic tipping point risk and imperfect substitutability has a surprisingly strong effect under plausible parameter settings on climate and the economy. Previous analytical modeling (36), although not specifically in a climate change context, has indicated that an environmental good that has limited substitutability with the aggregate consumption good, and is subject to risk, can exhibit a significantly lower discount rate than that for the consumption good, which would, in turn, favor environmental protection.

Our model illustration shows that estimates of optimal climate policy could dramatically change with the inclusion of environmental tipping points that impact nonmarket services that are difficult to substitute (18, 19). The effect of stochastic tipping points on substitutable market goods has previously been shown to yield a similar precautionary response (e.g., refs. 25 and 26). Here, combining the market and nonmarket effects of environmental tipping points (which is probably the most realistic case) yields an even stronger precautionary response. Stochastic tipping points (25, 26), and relative price effects (18, 19), are missing from many existing IAM studies, which consequently may be significantly underestimating the needs for controlling climate change. Our study highlights the importance of gaining more knowledge about environmental tipping points as well as the substitutability of environmental goods and services. The need for empirical research on substitutability of ecosystem services is recognized, but little research has been conducted yet (35). Our results might still be too optimistic compared with actual climate change, because here we consider cases in which the economy faces only a single tipping point. In reality, ecosystem risks exist at many levels and in many forms; therefore, multiple tipping points are likely to exist (1, 7).

Materials and Methods

For computations, we use a version of DSICE (27), a stochastic IAM of climate and the economy. DSICE is a stochastic extension of the annualized DICE model (28). The DICE model has been applied in numerous works, and the main drivers of its behavior have been studied extensively. Besides those associated with risk and uncertainty, the model parameters used for our analysis are calibrated the same as in DICE. DSICE computes the time paths of the optimal carbons emission control for the world. The global economy (as a single agent) is set to weigh the costs and benefits of emission control. Uncertainty (stochasticity) of climate change effects is included in such a way that the global economy makes emission control decisions by projecting future developments of climate and the economy that are not precisely known at the times of decisions. The model computes the time paths of the optimal allocation of emission reduction, investment in reproducible capital, and current consumption, which maximize the expected present value of global social welfare. The economic output is represented by a production function of capital and labor; the climate change damage fractionally reduces the gross economic output. The model includes the tipping point property for the environmental good determined by a Markov process; the risk aversion tendency built in to the social welfare function enhances emission reduction efforts to delay an uncertain permanent loss of the environmental good. See *SI Text* for a detailed description of the model.

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